

Supersymmetry Breaking and Fermi Balls

J.R. Morris

*Physics Department, Indiana University Northwest, 3400 Broadway, Gary,
Indiana 46408*

D. Bazeia

*Departamento de Física, Universidade Federal da Paraíba,
Caixa Postal 5008, 58051-970 João Pessoa, Paraíba, Brazil*

A simple model is presented where the disappearance of domain walls and the associated production of “Fermi balls”, which have been proposed as candidates for cold dark matter, are features which arise rather naturally in response to softly broken supersymmetry.

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I. INTRODUCTION

The possibility exists that the early universe experienced a sequence of symmetry breaking phase transitions, which could have resulted in the production of defects, such as monopoles, cosmic strings, or domain walls [1–3]. Furthermore, it is possible that supersymmetry could have been physically realized during early epochs, becoming broken after defect formation, at a lower energy scale. It is therefore quite natural to investigate models of defects within a supersymmetric context. Here, attention is focused upon a simple supersymmetric model constructed from a single chiral supermultiplet, which admits a domain wall solution interpolating between two distinct, but energetically degenerate, supersymmetric vacuum states. The domain wall formation arises from an exact discrete symmetry which is spontaneously broken. The initial supersymmetry of the model couples the fermion fields to the scalar fields in a prescribed way, and it is found that, as a result of this coupling, a fermion zero mode [4] forms within the core of the domain wall, where the fermion is essentially massless. Bound states can also exist which describe scalar bosons attached to the wall.

The breaking of supersymmetry at lower energies can be described by the inclusion of soft supersymmetry breaking terms in the scalar potential. However, it is found that when the soft supersymmetry breaking terms are added to the Lagrangian, the *exact* discrete symmetry responsible for the formation of the stable domain wall is transformed into an *approximate*, or *biased*, discrete symmetry. This approximate discrete symmetry results in a domain wall network where each wall interpolates between two different, energetically nondegenerate (and nonsupersymmetric) vacuum

states – a true vacuum state and a higher energy density false vacuum state [5,6]. Thus, the terms that are added to the Lagrangian to break the supersymmetry also explicitly break the exact discrete symmetry. The model then resembles one recently proposed by Macpherson and Campbell (MC) [7] wherein a biased discrete symmetry breaking results in the production of “Fermi balls” – tiny bags of false vacuum that are inhabited by a stabilizing Fermi gas. The Fermi balls emerge as an end product of the collapse and fragmentation of domain walls enclosing false vacuum protodomains. Massive, electrically neutral Fermi balls can be considered as candidates for cold dark matter. The simple model presented here therefore connects, in a rather natural way, the transformation of domain walls into Fermi balls and the breaking of supersymmetry.

The supersymmetric model is presented in the next section, where a domain wall solution is found. Upon examining the response of the fermion and boson fields to the domain wall background, it is seen that a fermion zero mode forms inside the wall, and that there can exist bound states describing the attachment of bosons to the wall. The fermion mass vanishes in the core of the wall, and fermions near the wall can experience a short ranged, but strong, attractive force toward the core of the wall where it is energetically more favorable for them to reside. This effect will be of importance in the consideration of Fermi balls, since the wall will tend to absorb fermions to become populated by a Fermi gas of effectively massless fermions that can contribute a fermion degeneracy pressure. The soft supersymmetry breaking terms that are to be added to the Lagrangian are given in sec. III. It is easily seen that the inclusion of these terms causes the previous exact discrete symmetry associated with the formation of the domain wall to be exchanged for an approximate, biased discrete symmetry. The basic mechanisms proposed by MC relating to the disappearance of the domain walls through the formation of false vacuum bags, the collapse and fragmentation of the vacuum bags, and the production of Fermi balls are briefly reviewed. It is seen that the model presented here, with the breaking of supersymmetry, closely resembles the model presented for Fermi balls, allowing an inference that, at least in the context of the simple model presented here, the production of Fermi balls can arise from the breaking of supersymmetry and a discrete symmetry. A short summary forms sec. IV.

II. THE SUPERSYMMETRIC MODEL

Consider a supersymmetric model constructed from a single chiral superfield Φ with component fields (ϕ, ψ, F) , where F represents the auxiliary boson field. The boson fields ϕ and F are complex scalar fields and the fermion field ψ is a Weyl two-spinor. Let us write the scalar field ϕ in the form $\phi = A + iB$, where A and B are real-valued. The superfield Φ has a superspace representation [8,9] given by

$$\Phi(y, \theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y), \quad (1)$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ and $\theta^2 = \theta\theta = \theta^\alpha\theta_\alpha$, $\alpha = 1, 2$. (We also have $\theta\psi = \theta^\alpha\psi_\alpha$, $\bar{\theta}\bar{\psi} = \bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}$, $\dot{\alpha} = 1, 2$.) A metric $g_{\mu\nu}$ with signature $(+, -, -, -)$ is used. (See the Appendix for a brief description of the conventions and gamma matrices.) One can also define a Majorana 4-spinor Ψ in terms of the Weyl 2-spinors:

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad \alpha = 1, 2, \quad \dot{\alpha} = 1, 2. \quad (2)$$

A. Lagrangian

In terms of the chiral superfield the Lagrangian can be written as

$$L = (\bar{\Phi}\Phi)|_{\theta^2\bar{\theta}^2} + W(\Phi)|_{\theta^2} + \bar{W}(\bar{\Phi})|_{\bar{\theta}^2}, \quad (3)$$

where $\bar{\Phi} = \Phi^*$, i.e., the “bar” and “star” symbols mean complex conjugation, $W(\Phi)$ is the superpotential, which will be defined shortly, and $X|_{\theta^2}$ stands for the θ^2 part of X , etc. By eliminating the auxiliary field F , the Lagrangian can be written in terms of the component fields as

$$L = L_K^B + L_K^F + L_Y - V, \quad (4)$$

where

$$L_K^B = \partial^\mu \bar{\phi} \partial_\mu \phi = \partial^\mu A \partial_\mu A + \partial^\mu B \partial_\mu B, \quad \phi = A + iB, \quad (5)$$

$$L_K^F = \frac{i}{2} [(\partial_\mu \psi) \sigma^\mu \bar{\psi} - \psi \sigma^\mu \partial_\mu \bar{\psi}] = \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi, \quad (6)$$

$$L_Y = -\frac{1}{2} \left[\left(\frac{\partial^2 W}{\partial \phi^2} \right) \psi \psi + \left(\frac{\partial^2 W}{\partial \phi^2} \right)^* \bar{\psi} \bar{\psi} \right], \quad (7)$$

$$V = |F|^2 = \left| \frac{\partial W}{\partial \phi} \right|^2, \quad F = - \left(\frac{\partial W}{\partial \phi} \right)^*. \quad (8)$$

B. Superpotential and Scalar Potential

To get a domain wall solution, let us choose the superpotential

$$W = \lambda \Phi \left(\frac{1}{3} \Phi^2 - a^2 \right) \quad (9)$$

so that the auxiliary field is described by $F^* = -\lambda(\phi^2 - a^2)$, where a is a constant. From (8) the scalar potential is then given by

$$V = F^* F = \lambda^2 (\bar{\phi}^2 - a^2)(\phi^2 - a^2) = \lambda^2 [(\bar{\phi}\phi)^2 - a^2(\phi^2 + \bar{\phi}^2) + a^4]. \quad (10)$$

The scalar potential $V = |F|^2 \geq 0$ has minima located at $F = 0$, which implies that the (supersymmetric) vacuum states of the theory are located by $\phi = \pm a$, i.e., the vacuum states of the theory are described by $A = \pm a$, $B = 0$. It is seen that supersymmetry is respected in the vacuum, where $V = 0$.

C. Field Equations

Recalling that $\phi = A + iB$, along with the expression for the Majorana spinor Ψ given by (2), the Lagrangian, expressed in terms of the real scalar fields A and B and the Majorana 4-spinor Ψ , can be written in the form given by (4) with (using $\partial^2 W / \partial \phi^2 = 2\lambda\phi$)

$$L = \partial^\mu A \partial_\mu A + \partial^\mu B \partial_\mu B + \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi + i\lambda [A \bar{\Psi} \Psi + B \bar{\Psi} \gamma_5 \Psi] - \lambda^2 [(A^2 - a^2)^2 + 2B^2(A^2 + a^2) + B^4], \quad (11)$$

where $(\psi\psi + \bar{\psi}\bar{\psi}) = -i\bar{\Psi}\Psi$ and $(\psi\psi - \bar{\psi}\bar{\psi}) = -\bar{\Psi}\gamma_5\Psi$ have been used. With $L = L_K^B + L_K^F + L_Y - V$, the field equations for A , B , and Ψ follow from

$$2\Box A + \frac{\partial V}{\partial A} - \frac{\partial L_Y}{\partial A} = 0, \quad (12)$$

$$2\Box B + \frac{\partial V}{\partial B} - \frac{\partial L_Y}{\partial B} = 0, \quad (13)$$

$$\frac{\partial L}{\partial \bar{\Psi}} = \frac{\partial L_K^F}{\partial \bar{\Psi}} + \frac{\partial L_Y}{\partial \bar{\Psi}} = 0. \quad (14)$$

Therefore, from (11)-(14), the field equations for A , B , and Ψ are given by

$$\Box A + 2\lambda^2 A(A^2 + B^2 - a^2) - i\frac{\lambda}{2} \bar{\Psi} \Psi = 0, \quad (15)$$

$$\Box B + 2\lambda^2 B(A^2 + B^2 + a^2) - i\frac{\lambda}{2} \bar{\Psi} \gamma_5 \Psi = 0, \quad (16)$$

$$\gamma^\mu \partial_\mu \Psi + 2\lambda(A + B\gamma_5)\Psi = 0, \quad (17)$$

where $\Box = \partial^\mu \partial_\mu$.

D. The Domain Wall and Particle Masses

Let us consider the real bosonic sector of the model where the fermionic fields vanish and the scalar field is real-valued, i.e. $\Psi = 0$, $B = 0$. As boundary conditions for the A field we take $A(x = \pm\infty) = \pm a$. Then (15) gives

$$\Box A + 2\lambda^2 A(A^2 - a^2) = 0, \quad (18)$$

which has a static solution

$$A_W(x) = a \tanh \frac{x}{w}, \quad w = \frac{1}{\lambda a}. \quad (19)$$

The solution given by (19) describes an ordinary domain wall of thickness $w = \frac{1}{\lambda a}$. We can notice that the domain wall can be associated with the spontaneous breaking of an exact discrete Z_2 symmetry describing invariance of the real bosonic Lagrangian under $A \rightarrow -A$. Using this domain wall solution as a background solution, the response of the fields Ψ and B can be examined. It will be seen that there is a fermionic zero mode within the domain wall, and that there are domain wall- B particle bound states.

In the vacuum states we have $A = \pm a$, $B = 0$, $\Psi = 0$. Therefore, in vacuum the B particle mass is determined to be $m_B = 2\lambda a$, and for the Majorana fermion, $L_Y^{(+a)} = i\lambda a \bar{\Psi}\Psi = \frac{i}{2}m_F \bar{\Psi}\Psi$, which gives $m_F = 2\lambda a = m_B$, which is expected from supersymmetry in the vacuum. (For the vacuum state $A = -a$, the mass eigenstate Weyl spinors must be redefined by a phase rotation, and the Majorana spinor undergoes a γ_5 rotation, $\Psi \rightarrow \gamma_5 \Psi$.)

In the core of the domain wall, $A \rightarrow 0$, and we find $m_F = 0$, $m_B = \sqrt{2}\lambda a$, so that the mass of each particle decreases within the domain wall. On this basis, we see that the particles are attracted toward the wall with a force $F \sim -\partial m(x)/\partial x$. The existence of scalar bound states and spinor zero modes is consistent with this picture. For the Majorana fermion we have $m_F(x) = 2\lambda A(x)$ (for $x > 0$, e.g.) and therefore, by (19), the force of attraction can be estimated to be $F \sim -\frac{2}{w^2} \left[\cosh \frac{x}{w} \right]^{-2}$, which, for a thin wall, can be quite large in magnitude (but of short range, rapidly vanishing outside the wall's surface).

E. Fermionic Zero Mode

1. Static Zero Mode

Upon setting $A = A_W(x)$, $B = 0$, the field equation for Ψ reduces to

$$\gamma^\mu \partial_\mu \Psi + 2\lambda A_W \Psi = 0. \quad (20)$$

For the gamma matrices we have $\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}$, $\gamma^1 = i \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}$, $(\gamma^1)^2 = 1$.

Let us first look for a static solution of the form $\Psi = \Psi(x)$. Multiplying (20) by γ^1 gives

$$\partial_x \Psi = -2\lambda A_W \gamma^1 \Psi. \quad (21)$$

Let us now write the Majorana 4-spinor Ψ in terms of 2-spinors η and χ : $\Psi = \begin{pmatrix} \eta \\ \chi \end{pmatrix}$.

We then have $\gamma^1 \Psi = i \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix} \begin{pmatrix} \eta \\ \chi \end{pmatrix} = i \begin{pmatrix} \sigma_1 \chi \\ -\sigma_1 \eta \end{pmatrix}$. Therefore,

$$\partial_x \begin{pmatrix} \eta \\ \chi \end{pmatrix} = -2i\lambda A_W \begin{pmatrix} \sigma_1 \chi \\ -\sigma_1 \eta \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (\sigma_1)^2 = 1. \quad (22)$$

The equations for η and χ can be decoupled by writing

$$\chi = -i\sigma_1 \eta, \quad \eta = i\sigma_1 \chi. \quad (23)$$

Then, by (22) and (23),

$$\partial_x \eta = -2\lambda A_W \eta, \quad \partial_x \chi = -2\lambda A_W \chi, \quad \Psi = \begin{pmatrix} \eta \\ \chi \end{pmatrix} = \begin{pmatrix} \eta \\ -i\sigma_1 \eta \end{pmatrix}. \quad (24)$$

A solution is given by

$$\eta = \tau \exp \left[-2\lambda \int_0^x A_W(x') dx' \right] = \tau \left[\cosh \frac{x}{w} \right]^{-2}, \quad (25)$$

where τ is an arbitrary constant Weyl 2-spinor.

The Majorana condition $\Psi_C = -\gamma^2 \Psi^* = \Psi$, (where Ψ_C is the charge conjugate of Ψ) i.e.

$$\Psi = \begin{pmatrix} \eta \\ \chi \end{pmatrix} = \begin{pmatrix} \eta \\ i\sigma_2 \eta^* \end{pmatrix}, \quad (26)$$

must also be satisfied. Upon comparing (24) and (26), we have $\sigma_2 \eta^* = -\sigma_1 \eta$, or $\sigma_1 \sigma_2 \eta^* = -\eta$, so that with the help of $\sigma_1 \sigma_2 = i\sigma_3$, we get $\eta^* = i\sigma_3 \eta$. We must therefore require that $\tau^* = i\sigma_3 \tau$.

2. Traveling Waves

Let us now regard Ψ to be a function of x , z , and t , i.e., $\Psi(x, z, t) = \begin{pmatrix} \eta(x, z, t) \\ -i\sigma_1 \eta(x, z, t) \end{pmatrix}$, where $\eta(x, z, t) = \tau(z, t) \left[\cosh \frac{x}{w} \right]^{-2}$. Then (20) implies that

$$(\gamma^0 \partial_0 + \gamma_3 \partial_3) \begin{pmatrix} \tau(z, t) \\ -i\sigma_1 \tau(z, t) \end{pmatrix} \left[\cosh \frac{x}{w} \right]^{-2} = 0, \quad (27)$$

which is solved by

$$(\partial_0 - \sigma_3 \partial_3) \tau(z, t) = 0. \quad (28)$$

This can be seen by multiplying (27) by γ^1 and using $\gamma^0 \gamma^3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$, so that (27) reduces to the set of equations $(\partial_0 - \sigma_3 \partial_3) \tau = 0$, and $(\partial_0 + \sigma_3 \partial_3) \sigma_1 \tau = 0$, and

the second equation is automatically solved when the first equation is solved. Then (28) can be written explicitly as

$$\begin{pmatrix} (\partial_0 - \partial_3) & 0 \\ 0 & (\partial_0 + \partial_3) \end{pmatrix} \begin{pmatrix} \tau_1(z, t) \\ \tau_2(z, t) \end{pmatrix} = 0. \quad (29)$$

This is solved by

$$\tau_1(z, t) = \tau_1(z + t), \quad \tau_2(z, t) = \tau_2(z - t). \quad (30)$$

Therefore, τ can be written as

$$\tau(z, t) = \begin{pmatrix} \tau_1(z + t) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \tau_2(z - t) \end{pmatrix}, \quad (31)$$

so that τ , and hence Ψ , can contain a linear combination of “up” and “down” moving waves.

F. B Particle Bound States

Now let us set Ψ equal to zero and examine the B field in the domain wall background. From the field equation for B , we have

$$\square B + 2\lambda^2 B(A_W^2 + B^2 + a^2) = 0. \quad (32)$$

Now linearize, and look at small fluctuations about $B = 0$ to obtain

$$\square B + 2\lambda^2 a^2 \left(1 + \tanh^2 \frac{x}{w}\right) B = 0. \quad (33)$$

Writing $B(x, z, t) = b(x) \sin(kz - \omega t + \delta)$, (33) reduces to

$$-\partial_x^2 b + 2\lambda^2 a^2 [\tanh^2(x/w)] b = E^2 b, \quad E^2 \equiv \omega^2 - (k^2 + 2\lambda^2 a^2). \quad (34)$$

This is a Schrodinger-like equation with an attractive potential that can accommodate one or more bound states [10] with $0 < E < \sqrt{2}\lambda a$. We therefore infer that real scalar B particles can be localized within or near the core of the domain wall. For $E > \sqrt{2}\lambda a$ there can exist a set of states describing the scattering of B particles from the domain wall.

III. SOFT SUPERSYMMETRY BREAKING AND FERMI BALLS

The supersymmetry that exists in the vacuum states of the above model can be broken by adding soft supersymmetry breaking terms to the scalar potential [11]. The types of soft terms allowed here include a scalar mass term of the form $\mu^2 \bar{\phi} \phi$ and a trilinear scalar interaction of the form $[W(\Phi)|_{\theta=0} + c.c.] = g_0[\phi^3 + \bar{\phi}^3]$. Let us also add a (dynamically irrelevant) constant V_0 and therefore define the potential term

$$\begin{aligned} V_B &= \mu^2 \bar{\phi} \phi + g_0 (\phi^3 + \bar{\phi}^3) + V_0 \\ &= \mu^2 (A^2 + B^2) + g_0 (A^3 - 3AB^2) + V_0, \end{aligned} \quad (35)$$

where the constant V_0 can be used to adjust the vacuum energy of the true vacuum state to zero. The total potential can be written as

$$V_1 = V + V_B. \quad (36)$$

Notice that not only has the original supersymmetry been broken, but the discrete Z_2 symmetry associated with the reflection $A \rightarrow -A$ has also been broken by V_B . In fact, the model now resembles the kind of model [7] that was introduced for the description of a *Fermi ball*. Therefore, we have the possibility that a simple supersymmetric model, which possesses an exact discrete symmetry before the breaking of supersymmetry, can be left with an approximate discrete symmetry after the breaking of the supersymmetry, so that energetically nondegenerate vacuum states develop – a true vacuum state and a higher energy false vacuum state. Two different domains are separated by a domain wall, which can rapidly fragment into a mist of Fermi balls by the mechanism described by MacPherson and Campbell (MC). Massive neutral Fermi balls which couple only weakly with ordinary matter are then candidates for cold dark matter. Let us briefly review the Fermi ball model proposed by MC and then examine the model presented here to see how it can describe Fermi balls.

A. Fermi Balls

The basic scenario described by MC for the production of Fermi balls can be briefly summarized in the following. (For discussions of biased discrete symmetry breaking and domain walls, see also refs. [5,6].) Consider a simple model of a self-interacting scalar field φ and a Dirac fermion field ψ which is strongly coupled to the scalar field φ . The system can be described as follows: first consider the Lagrangian

$$L_0 = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - \frac{\lambda^2}{8} (\varphi^2 - \varphi_0^2)^2. \quad (37)$$

L_0 possesses a discrete Z_2 reflection symmetry, and the degenerate vacuum states of the theory are described by $\varphi = \pm \varphi_0$. A domain wall solution of the form $\varphi =$

$\varphi_0 \tanh\left(\frac{x}{\delta}\right)$ interpolates between the two distinct vacuum states, with $\varphi \rightarrow \pm\varphi_0$ as $x \rightarrow \pm\infty$, and in the core of the domain wall $\varphi \rightarrow 0$. The surface tension is equal to the surface energy density, given by

$$\sigma = \frac{2\lambda\varphi_0^3}{3}. \quad (38)$$

Let us now consider changing the exact discrete Z_2 symmetry to an approximate symmetry. The symmetry breaking results in the formation of two different, nondegenerate, vacuum states that form protodomains of true and false vacuum. Let the difference in the energy densities of the two vacuum states be represented by Λ . In this biased discrete symmetry breaking a domain wall can form which interpolates between the true and false vacuum protodomains. The exact Z_2 symmetry can be exchanged for an approximate symmetry by adding, for example, a Z_2 symmetry breaking term $A(\varphi)$ to the Lagrangian. The asymmetry that is introduced can result in the formation of finite sized “false vacuum bags” – regions of false vacuum protodomain enclosed by domain wall [5,6]. These vacuum bags can collapse, and result in the conversion of false vacuum into true vacuum and the disappearance of the domain walls.

Let us now consider a Dirac fermion ψ that is strongly coupled to the scalar field φ through a standard Yukawa coupling. We replace the Lagrangian L_0 with

$$L = \bar{\psi}(i\gamma^\mu\partial_\mu + iG\varphi)\psi + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{\lambda^2}{8}(\varphi^2 - \varphi_0^2)^2 + A(\varphi). \quad (39)$$

[A relative factor of $(-i)$ appears in the Yukawa term in (39) due to our choice of representation for the gamma matrices.] The fermion acquires different masses in the different protodomains due to its coupling with φ , but the fermion becomes effectively massless in the core of the domain wall where $\varphi \rightarrow 0$. It is therefore energetically favorable for the fermion to reside inside the wall, and it is assumed that fermions near the wall will become absorbed by the wall, so that the wall is quickly populated by massless fermions. As an estimate, we can think of a force of attraction acting on the fermions by the wall to be given roughly by $f(x) \sim -\partial M(x)/\partial x \sim -G\partial\varphi(x)/\partial x$. In the thin wall approximation, we can then regard the domain wall as possessing a two dimensional Fermi gas of massless fermions. The Fermi gas pressure can have a stabilizing influence by counteracting the wall surface tension and false vacuum pressure contributions. For a spherical vacuum bag, the collapse can be halted when the bag has a radius R , which minimizes the total energy E , and is related to the number of fermions N inhabiting the wall.

However, a vacuum bag of energy E and radius R is not stable against flattening into a “pancake” shape. The tendency to flatten thus results in the fragmentation of the vacuum bag into many smaller ones. The fragmentation process halts when the thin wall approximation is no longer valid, i.e. when the typical radius of curvature of a bag becomes comparable to the wall thickness. In this limit, the configuration

is better represented by a “Fermi ball” which can be thought of as a tiny vacuum bag with essentially no false vacuum in the interior - a nontopological scalar field configuration consisting mostly of the domain wall inhabited by a three dimensional Fermi gas. By equating the minimum size of the stabilized Fermi ball R_{\min} to the wall thickness δ , and assuming the stable Fermi ball to be spherical, the typical stabilizing radius at which the collapse and fragmentation process stops is estimated to be

$$R_{\min} \sim \frac{2}{\lambda\varphi_0}. \quad (40)$$

The presence of a Fermi gas inside the domain wall is crucial to the formation of Fermi balls in this model. In order that fermions and antifermions do not undergo annihilation processes that leave no Fermi gas inside the wall, it is sufficient to assume that there is a net fermion antifermion asymmetry, so that annihilation processes which may occur will eventually stop when all of the antifermions (or fermions) have been consumed, leaving a Fermi gas of fermions (or antifermions).

MC estimate that a Fermi ball would contain about 50 fermions and have a mass of roughly $100\varphi_0$ GeV, where φ_0 is expressed in units of GeV. If the Fermi balls are constructed from a new Dirac fermion that has no standard model gauge charges, then the Fermi ball would likely be a neutral, heavy, nonrelativistic particle interacting only very weakly with ordinary matter. In this case, Fermi balls would form a candidate for cold dark matter.

B. Supersymmetry Breaking and Fermi Balls

In the model presented here, the soft supersymmetry breaking terms are embedded in the potential term V_B , given by (35). Thus, by (11) and (35) the total Lagrangian, written in terms of the real scalar fields A and B and the Majorana field Ψ , is given by

$$\begin{aligned} L_1 = & \partial^\mu A \partial_\mu A + \partial^\mu B \partial_\mu B + \frac{i}{2} \bar{\Psi} \gamma^\mu \partial_\mu \Psi + i\lambda[A\bar{\Psi}\Psi + B\bar{\Psi}\gamma_5\Psi] \\ & -\lambda^2[(A^2 - a^2)^2 + 2B^2(A^2 + a^2) + B^4] \\ & -\mu^2(A^2 + B^2) - g_0(A^3 - 3AB^2) - V_0. \end{aligned} \quad (41)$$

This model closely resembles that presented by MC, except that (41) contains an additional scalar field B and the fermion here is Majorana, rather than Dirac. [Let us also assume a parameter range that keeps the vacuum expectation value $B_{vac} = 0$ after supersymmetry breaking. For example, we could require that $\frac{\mu^2}{2\lambda^2} \ll a^2$, $\frac{3g_0}{2\lambda^2} \ll a$, and $\mu^2 - 3g_0a > 0$, so that after the breaking of the supersymmetry $B_{vac} = 0$ and the vacuum values of A are only slightly shifted from $\pm a$.] Upon setting $B = 0$, (41) is seen to have the same form as (39). The supersymmetry breaking terms embedded in V_B also break the exact discrete symmetry associated with $A \rightarrow -A$ in the supersymmetric version. We therefore expect the Fermi ball

scenario to be realized in this broken supersymmetric model, as well. Following the reasoning of MC, we expect a Fermi gas to remain within the domain wall after possible fermion antifermion annihilations if there is a fermion antifermion asymmetry, or when it becomes energetically unfavorable for massive particles, like the A and B scalar bosons, to be produced outside the wall. The Fermi balls associated with this model are neutral, and again can be considered as candidates for cold dark matter. It can also be pointed out that a slightly more complicated model [12], composed of two interacting chiral superfields (and hence two Majorana fermion fields, or equivalently, a Dirac field), that has the same basic features presented here might be implemented. The Majorana fermion can then be replaced with a Dirac fermion. It is conceivable that a model of this type could be constructed where the scalar and spinor fields are allowed to have standard model gauge couplings. But one of the points to be illuminated in this work is that the MC type of model, which leads to the prediction of Fermi balls, can emerge rather naturally in an initially supersymmetric theory where the supersymmetry gets broken, along with the exact discrete symmetry. The breaking of the exact discrete symmetry can remove the potentially hazardous domain wall problem, and also give rise to the production of Fermi balls.

IV. SUMMARY

Because there exists a strong possibility that (1) the early universe underwent a set of symmetry breaking phase transitions, during which defects may have been formed, and (2) supersymmetry was physically realized at the time of defect production and was broken at a somewhat later time, it becomes relevant to consider field theoretic models of defects within the context of supersymmetry. Here, a simple domain wall model, constructed from a single chiral superfield, has been examined. It is found that the fermions become massless inside the domain wall, where a zero mode forms, and that there are bound states describing real scalar bosons attached to the wall. (These types of results have been examined previously for a supersymmetric model constructed from two interacting chiral superfields [12], but the effects of soft supersymmetry breaking terms are more easily examined in the single field model presented here.)

An exact Z_2 discrete symmetry in the bosonic sector of the theory, associated with the reflection symmetry $A \rightarrow -A$, gets broken explicitly to an approximate biased discrete symmetry when soft supersymmetry breaking terms are added to the Lagrangian. The model then closely resembles one describing “Fermi balls”, which are scalar field configurations stabilized by a Fermi gas exerting a degeneracy pressure. In the context of the simple model presented here, we therefore expect the production of domain walls, followed by a process wherein the walls form false vacuum bags (due to the transformation of the exact discrete symmetry into an approximate one), which collapse and fragment, finally resulting in the production of Fermi balls. Thus, in the model presented here, the production of Fermi balls is closely associated with the

breaking of supersymmetry.

APPENDIX A: CONVENTIONS

Some of the notations and conventions are briefly listed here. A metric $g_{\mu\nu}$ is used with signature $(+, -, -, -)$. Aside from the metric, the notation, conventions, and gamma matrices used conform to those of ref. [8]. The gamma matrices can be written in the form

$$\gamma^\mu = i \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad (\text{A1})$$

with

$$\sigma^\mu = (1, \vec{\sigma}) , \quad \bar{\sigma}^\mu = (1, -\vec{\sigma}) , \quad (\text{A2})$$

where $\vec{\sigma}$ represents the Pauli matrices. Then

$$\gamma^0 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^k = i \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \quad k = 1, 2, 3, \quad (\text{A3})$$

and γ_5 is given by

$$\gamma_5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3 = i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (\text{A4})$$

The gamma matrices have the properties

$$\{\gamma^\mu, \gamma^\nu\} = -2g^{\mu\nu}, \quad \{\gamma^\mu, \gamma_5\} = 0, \quad \gamma_5^\dagger = -\gamma_5, \quad (\gamma_5)^2 = -1. \quad (\text{A5})$$

A Majorana 4-spinor Ψ is expressed in terms of the Weyl 2-spinors ψ and $\bar{\psi}$ by $\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix}$ and we use the summation conventions for Weyl spinors [with $\bar{\psi}^{\dot{\alpha}} = (\psi^\alpha)^*$]

$$\xi\psi \equiv \xi^\alpha \psi_\alpha, \quad \bar{\xi}\bar{\psi} \equiv \bar{\xi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}, \quad \alpha = 1, 2, \quad \dot{\alpha} = 1, 2, \quad (\text{A6})$$

with ε metric tensors (for raising and lowering Weyl spinor indices)

$$(\varepsilon^{\alpha\beta}) = (\varepsilon^{\dot{\alpha}\dot{\beta}}) = i\sigma_2, \quad (\varepsilon_{\alpha\beta}) = (\varepsilon_{\dot{\alpha}\dot{\beta}}) = -i\sigma_2, \quad \varepsilon^{12} = 1 = \varepsilon^{\dot{1}\dot{2}}. \quad (\text{A7})$$

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